

Micro α^* -Generalized Separation Axioms In Micro Topological Spaces

K.Selvakumari Prasha^{1*}, Dr.P.Anbarasi Rodrigo²

^{1*}Research Scholar, Reg. No. 23112212092002, Department of Mathematics, St.Mary's College (Autonomous), (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627 012) Thoothukudi- 628 001, Tamilnadu, India.

²Assistant Professor, Department of Mathematics, St.Mary's College (Autonomous), (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627 012) Thoothukudi- 628 001, Tamilnadu, India. E-mail: anbu.n.u@gmail.com.

^{1*}Corresponding Author: selvakumarprasha12345@gmail.com.

Abstract

S.Chandrasekar introduced MT in 2019. We introduced a new class of M-Generalized C Sets in MTS. In this paper, we introduced some forms of separation axioms namely $\alpha^*g_M - T_0$ Space, $\alpha^*g_M - T_1$ Space and α^*g_M - Hausdroff Space. We study the connections of these functs with other forms of M-separation axioms. Also we established the relation between the above funct and also investigate various aspects of these functs. **Method:** The study employs rigorous analysis of these M-functs and formal proof technique to establish their properties. Comparative analysis with some existing funct is conducted to highlight the distinct characteristics of these axioms. **Finding:** The research identifies and confirms the unique features of $\alpha^*g_M - T_0$ Space, $\alpha^*g_M - T_1$ Space and α^*g_M - Hausdroff Space under various operations, supported by detailed proofs and examples. **Novelty:** The research contributes to the mathematical literature by defining and exploring a new category of M-funct and comparing it with some of the existing functs.

Keywords: α^*g_M -C, α^*g_M -O, $\alpha^*g_M - T_0$ Space, $\alpha^*g_M - T_1$ Space and α^*g_M - Hausdroff Space. **[2000] Mathematics Subject Classifications:** 54D10.

I: Introduction

Topology comprises one of the oldest and most significant topics of Mathematics; the idea of Topology is becoming vaster and more complex by the day. Several versions of Topologies have recently been developed until now. The concept of NT was introduced by Lellis Thivagar in 2013 which is defined in terms of the lower approx, higher approx, and the boundary area, subset of the universe set involving the

equivalence relation. The notation of approx and the boundary area of a set were originally introduced by Pawlak in order to introduce the concept of rough sets theory. MT was an expansion of NT. S.Chandrasekar introduced and researched M-Semi O, M-Pre O, M-Semi cont and M-Pre cont in MTS. A New Class of M-Generalized C Sets in MTS i.e. $\alpha^*g_{\mathcal{M}}$ -C Sets is introduced by P.Anbarasi Rodrigo and K.Selvakumari Prasha. The purpose of this paper was too introduced and analyzed the concepts of $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_0$ Space, $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space and $\alpha^*g_{\mathcal{M}}$ -Hausdroff Space in MTS.

II: Methodology

This paper introduces and extends the concepts of $\alpha^*g_{\mathcal{M}}$ -separation axioms within the frameworks of MTS. It explores the connection between the newly defined functs and the previously established M-Cont through the application of definitions. Additionally, the paper provides examples to demonstrate the converse parts of these relationships and other properties.

III: Preliminaries

In this portion, we review a few essential MTS definitions and outcomes that will be important in the sequel.

Definition: 3.1 Let $\mathbb{U} \neq \phi$, finite class of elements known as universe & $\mathcal{R}$ denotes the equivalence relation in $\mathbb{U}$ termed as indiscernibility relation. Therefore $\mathbb{U}$ is parted into distinct equivalence classes. Object from same equivalence class has been named as indiscernible with one another. And $(\mathbb{U}, \mathcal{R})$, the pair is approx space. Take $\mathbb{X} \subseteq_{\mathcal{M}} \mathbb{U}$.

1) The lower Approx of $\mathbb{X}$ in regards to $\mathcal{R}$ would be the class of entire elements, that could be for certain classified as $\mathbb{X}$ in terms of $\mathcal{R}$ which is notated as $\mathbb{L}_{\mathcal{R}}(\mathbb{X})$. i.e. $\mathbb{L}_{\mathcal{R}}(\mathbb{X}) = \bigcup_{\mathbb{X} \in \mathbb{U}} \{\mathcal{R}(\mathbb{X}) : \mathcal{R}(\mathbb{X}) \subseteq_{\mathcal{M}} \mathbb{X}\}$, here $\mathcal{R}(\mathbb{X})$ refer the equivalence class gained by $\mathbb{X} \in \mathbb{U}$.

2) The upper Approx of $\mathbb{X}$ under $\mathcal{R}$ would be the class of elements, which could be possibly characterized as $\mathbb{X}$ by $\mathcal{R}$ & notated by $\mathbb{U}_{\mathcal{R}}(\mathbb{X})$. i.e. $\mathbb{U}_{\mathcal{R}}(\mathbb{X}) = \bigcup_{\mathbb{X} \in \mathbb{U}} \{\mathcal{R}(\mathbb{X}) : \mathcal{R}(\mathbb{X}) \cap \mathbb{X} \neq \phi\}$.

3) The boundary area of $\mathbb{X}$ in regards to $\mathcal{R}$ is the class of elements, that could be specified neither as $\mathbb{X}$ nor as not $\mathbb{X}$ in regards with $\mathcal{R}$ & notated by $\mathbb{B}_{\mathcal{R}}(\mathbb{X})$. i.e. $\mathbb{B}_{\mathcal{R}}(\mathbb{X}) = \mathbb{U}_{\mathcal{R}}(\mathbb{X}) - \mathbb{L}_{\mathcal{R}}(\mathbb{X})$.

Definition: 3.2 Consider universe $(\mathbb{U}, \mathcal{R})$, equivalence relation in $\mathbb{U}$ & $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \mathbb{L}_{\mathcal{R}}(\mathbb{X}), \mathbb{U}_{\mathcal{R}}(\mathbb{X}), \mathbb{B}_{\mathcal{R}}(\mathbb{X})\}$, here $\mathbb{X} \subseteq_{\mathcal{M}} \mathbb{U}$. So that $\tau_{\mathcal{R}}(\mathbb{X})$ fulfills the below criteria:

- 1) $\mathbb{U} \& \phi \in \tau_{\mathcal{R}}(\mathbb{X})$.
- 2) The union of objects from the sub-classes of $\tau_{\mathcal{R}}(\mathbb{X})$ lies in $\tau_{\mathcal{R}}(\mathbb{X})$.
- 3) The intersection of object of any finite sub-classes of $\tau_{\mathcal{R}}(\mathbb{X})$ lies in $\tau_{\mathcal{R}}(\mathbb{X})$. i.e. $\tau_{\mathcal{R}}(\mathbb{X})$ is a

Topology in $\mathbb{U}$ said as NT on $\mathbb{U}$ in regards to $\mathbb{X}$. $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}))$ termed as Nano Topological Space. Object from $\tau_{\mathcal{R}}(\mathbb{X})$ are the Nano O sets and its complements is Nano C sets.

Definition: 3.3 [3] Consider $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}))$, an NTS. So $\mu_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{N} \cup (\mathbb{N}' \cap \mu) : \mathbb{N}, \mathbb{N}' \in \tau_{\mathcal{R}}(\mathbb{X})\}$ & it is referred as MT of $\tau_{\mathcal{R}}(\mathbb{X})$ and $\mu \notin \tau_{\mathcal{R}}$ meet the given axioms:

- 1) $\mathbb{U}, \phi \in \mu_{\mathcal{R}}(\mathbb{X})$.
- 2) The union of objects from the Sub-classes of $\mu_{\mathcal{R}}(\mathbb{X})$ lies in $\mu_{\mathcal{R}}(\mathbb{X})$.
- 3) The intersection of objects from any finite sub-classes of $\mu_{\mathcal{R}}(\mathbb{X})$ lies in $\mu_{\mathcal{R}}(\mathbb{X})$. So $\mu_{\mathcal{R}}(\mathbb{X})$

is said as the M-Topological on $\mathbb{U}$ in regards to $\mathbb{X}$. $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ is termed as MTS and objects from $\mu_{\mathcal{R}}(\mathbb{X})$ are the M-O & its complement is termed as a M-C sets.

Definition: 3.4 [1] Consider $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ a MTS. A subset $\mathbb{A}$ of $\mathbb{U}$ is said to be $\alpha^*g_{\mathcal{M}}$ -C set if $\mathcal{M}\alpha^*cl(\mathbb{A}) \subseteq_{\mathcal{M}} \text{int}\mathbb{U}$, whence $\mathbb{A} \subseteq_{\mathcal{M}} \mathbb{U}$ & $\mathbb{U}$ is a M- α -O in $\mathbb{U}$.

Example: 3.5 Take $\mathbb{U} = \{a, b, c, d\} \ni \mathbb{U}/\mathcal{R} = \{\{a\}, \{c\}, \{b, d\}\}$ and $\mathbb{X} = \{b, c\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{b, d\}\}$, $\mu = \{a\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-C} = \{\mathbb{U}, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

Result: 3.6 [1] Every $\mathcal{M}$ -C is $\alpha^*g_{\mathcal{M}}$ -C.

Definition: 3.7 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ and $(\mathbb{V}, \tau_{\mathcal{R}}^*(\mathbb{M}), \mu_{\mathcal{R}}^*(\mathbb{M}))$ be two MT.

- 1) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be $\alpha^*g_{\mathcal{M}}$ -cont funct if the inverse image of every $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{U}$ is M-C in $\mathbb{V}$.
- 2) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be $\alpha^*g_{\mathcal{M}}$ -irres funct if $\mathbb{F}^{-1}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{U}$, then $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{V}$.
- 3) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be strongly $\alpha^*g_{\mathcal{M}}$ -CM if for each $\alpha^*g_{\mathcal{M}}$ -C set $\mathbb{S}$ in $\mathbb{U}$, $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{V}$.
- 4) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be strongly $\alpha^*g_{\mathcal{M}}$ -OM if for each $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{S}$ in $\mathbb{U}$, $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}$ -O in $\mathbb{V}$.
- 5) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be $\alpha^*g_{\mathcal{M}}$ -CM if the image of every M-C set $\mathbb{S}$ in $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{V}$.
- 6) A funct $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is said to be $\alpha^*g_{\mathcal{M}}$ -OM if the image of every M-O set $\mathbb{S}$ in $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ -O in $\mathbb{V}$.
- 7) $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space or $\alpha^*g_{\mathcal{M}}$ -space if every $\alpha^*g_{\mathcal{M}}$ -C in $\mathbb{U}$ is M-C in $\mathbb{U}$.

Definition: 3.8 A $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ is a MTS $\mathbb{U}$ is said to be

- 1) $\mathcal{M}$ - $\mathcal{T}_0$ Space if for each pair of distinct points $\mathbb{S}$ and $\mathbb{T}$ of $\mathbb{U}$, there exists $\alpha^*g_{\mathcal{M}}$ -O sets $\mathbb{P}$ such that either $\mathbb{S} \in \mathbb{P}$ and $\mathbb{T} \notin \mathbb{P}$ or $\mathbb{S} \notin \mathbb{P}$ and $\mathbb{T} \in \mathbb{P}$.
- 2) $\mathcal{M}$ - $\mathcal{T}_1$ Space if for each pair of distinct points $\mathbb{S}$ and $\mathbb{T}$ of $\mathbb{U}$, there exists M-O sets $\mathbb{P}$ and $\mathbb{Q}$ such that one containing $\mathbb{S} \in \mathbb{P}$ but $\mathbb{T} \notin \mathbb{P}$ and $\mathbb{S} \notin \mathbb{P}$ but $\mathbb{T} \in \mathbb{P}$.
- 3) $\mathcal{M}$ - $\mathcal{T}_2$ Space (M-Hausdroff Space) if for each pair of distinct points $\mathbb{S}, \mathbb{T} \in \mathbb{U}$ and $\mathbb{S} \neq \mathbb{T}$, there exists M-O sets $\mathbb{P}$ and $\mathbb{Q}$ such that $\mathbb{P} \cap \mathbb{Q} = \phi$ and $\mathbb{S} \in \mathbb{P}, \mathbb{T} \in \mathbb{Q}$.

IV: $\alpha^*g_{\mathcal{M}}\text{-}\mathcal{T}_i$ ($i = 0, 1, 2$) Space

Definition: 4.1 A $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ is a MTS. Then $\mathbb{U}$ is said to be:

- 1) $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$ if for each pair of distinct points $\mathbb{S}$ and $\mathbb{T}$ of $\mathbb{U}$, there exists $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P}$ such that either $\mathbb{S} \in \mathbb{P}$ and $\mathbb{T} \notin \mathbb{P}$ or $\mathbb{S} \notin \mathbb{P}$ and $\mathbb{T} \in \mathbb{P}$.
- 2) $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1\text{Space}$ if for each pair of distinct points $\mathbb{S}$ and $\mathbb{T}$ of $\mathbb{U}$, there exists $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P}$ and $\mathbb{Q}$ such that one containing $\mathbb{S} \in \mathbb{P}$ but $\mathbb{T} \notin \mathbb{P}$ and $\mathbb{S} \notin \mathbb{P}$ but $\mathbb{T} \in \mathbb{P}$.
- 3) $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2\text{Space}$ ($\alpha^*g_{\mathcal{M}}\text{-Hausdroff Space}$) if for each pair of distinct points $\mathbb{S}, \mathbb{T} \in \mathbb{U}$ and $\mathbb{S} \neq \mathbb{T}$, there exists $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P}$ and $\mathbb{Q}$ such that $\mathbb{P} \cap \mathbb{Q} = \emptyset$ and $\mathbb{S} \in \mathbb{P}$, $\mathbb{T} \in \mathbb{Q}$.

Instance: 4.2

- 1) Take $\mathbb{U} = \{\mathbb{a}, \mathbb{b}, \mathbb{c}\} \ni \mathbb{U}/\mathcal{R} = \{\{\mathbb{a}\}, \{\mathbb{c}, \mathbb{b}\}\}$ and $\mathbb{X} = \{\mathbb{b}\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}, \mathbb{c}\}\}$, $\mu = \{\mathbb{a}, \mathbb{b}\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}\}, \{\mathbb{a}, \mathbb{b}\}, \{\mathbb{b}, \mathbb{c}\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-}$ O($\mathbb{U}$) = $\{\mathbb{U}, \phi, \{\mathbb{a}\}, \{\mathbb{b}\}, \{\mathbb{c}\}, \{\mathbb{a}, \mathbb{c}\}, \{\mathbb{a}, \mathbb{b}\}, \{\mathbb{b}, \mathbb{c}\}\}$. Then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, since $\{\mathbb{b}, \mathbb{c}\} \in \mathbb{U}$ implies that there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\{\mathbb{a}, \mathbb{b}\}$ such that $\{\mathbb{b}\} \in \{\mathbb{a}, \mathbb{b}\}$ but $\{\mathbb{c}\} \notin \{\mathbb{a}, \mathbb{b}\}$. Take $\{\mathbb{a}, \mathbb{c}\} \in \mathbb{U}$. Then there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\{\mathbb{b}, \mathbb{c}\}$ such that $\{\mathbb{c}\} \in \{\mathbb{b}, \mathbb{c}\}$ but $\{\mathbb{a}\} \notin \{\mathbb{b}, \mathbb{c}\}$. If we take $\{\mathbb{a}, \mathbb{b}\} \in \mathbb{U}$. Then there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\{\mathbb{b}, \mathbb{c}\}$ such that $\{\mathbb{b}\} \in \{\mathbb{b}, \mathbb{c}\}$ but $\{\mathbb{a}\} \notin \{\mathbb{b}, \mathbb{c}\}$.
- 2) Take $\mathbb{U} = \{\mathbb{a}, \mathbb{b}, \mathbb{c}\} \ni \mathbb{U}/\mathcal{R} = \{\{\mathbb{a}\}, \{\mathbb{c}, \mathbb{b}\}\}$ and $\mathbb{X} = \{\mathbb{b}\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}, \mathbb{c}\}\}$, $\mu = \{\mathbb{a}, \mathbb{b}\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}\}, \{\mathbb{a}, \mathbb{b}\}, \{\mathbb{b}, \mathbb{c}\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-}$ O($\mathbb{U}$) = $\{\mathbb{U}, \phi, \{\mathbb{a}\}, \{\mathbb{b}\}, \{\mathbb{c}\}, \{\mathbb{a}, \mathbb{c}\}, \{\mathbb{a}, \mathbb{b}\}, \{\mathbb{b}, \mathbb{c}\}\}$. Then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1\text{Space}$, since $\{\mathbb{b}, \mathbb{c}\} \in \mathbb{U}$ implies that there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\{\mathbb{a}, \mathbb{b}\}$ such that $\{\mathbb{b}\} \in \{\mathbb{a}, \mathbb{b}\}$ but $\{\mathbb{c}\} \notin \{\mathbb{a}, \mathbb{b}\}$. Take $\{\mathbb{a}, \mathbb{c}\} \in \mathbb{U}$. Then there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\{\mathbb{b}, \mathbb{c}\}$ such that $\{\mathbb{c}\} \in \{\mathbb{b}, \mathbb{c}\}$ but $\{\mathbb{a}\} \notin \{\mathbb{b}, \mathbb{c}\}$.
- 3) Take $\mathbb{U} = \{\mathbb{a}, \mathbb{b}, \mathbb{c}, \mathbb{d}\} \ni \mathbb{U}/\mathcal{R} = \{\{\mathbb{a}, \mathbb{c}, \mathbb{d}\}, \{\mathbb{b}\}\}$ and $\mathbb{X} = \{\mathbb{b}\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}\}\}$, $\mu = \{\mathbb{a}\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \{\mathbb{b}\}, \{\mathbb{a}, \mathbb{c}\}, \{\mathbb{a}, \mathbb{b}, \mathbb{c}\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-}$ O = $\{\mathbb{U}, \phi, \{\mathbb{b}\}, \{\mathbb{a}, \mathbb{c}\}, \{\mathbb{b}, \mathbb{d}\}, \{\mathbb{a}, \mathbb{b}, \mathbb{c}\}, \{\mathbb{a}, \mathbb{c}, \mathbb{d}\}\}$. Let $\mathbb{S} = \{\mathbb{a}\}$ and $\mathbb{T} = \{\mathbb{b}\}$ are disjoint points in $\mathbb{U}$. Then there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P} = \{\mathbb{a}, \mathbb{c}\}$ and $\mathbb{Q} = \{\mathbb{b}\}$ such that $\mathbb{S} \in \mathbb{P}$, $\mathbb{T} \in \mathbb{Q}$.

Theorem: 4.3 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) Every $\mathcal{M}\text{-}\mathbb{T}_0\text{Space}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.
- 2) Every $\mathcal{M}\text{-}\mathbb{T}_1\text{Space}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1\text{Space}$.
- 3) Every $\mathcal{M}\text{-}\mathbb{T}_2$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2\text{Space}$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) Then each pair of distinct points x and y of $\mathbb{U}$, there exists a M-O set $\mathbb{P}$ in $\mathbb{U}$ containing one point but not the other. Then $\mathbb{P}$ is M-O set in $\mathbb{U}$ because every M-O is a $\alpha^*g_{\mathcal{M}}\text{-}$ O, $\mathbb{P}$ is $\alpha^*g_{\mathcal{M}}\text{-}$ O sets in $\mathbb{U}$. Also $x \in \mathbb{P}$ and $y \notin \mathbb{P}$. Therefore $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.
- 2) Then each pair of distinct points x and y of $\mathbb{U}$, then there exists a M-O sets $\mathbb{P}$ and $\mathbb{Q}$ such that one containing $\mathbb{S}$ but not $\mathbb{T}$ and the other containing $\mathbb{T}$ but not $\mathbb{S}$. That is $x \in \mathbb{P}$ but $y \notin \mathbb{P}$ and $y \in \mathbb{Q}$ but $x \notin \mathbb{Q}$. Since every M-O is $\alpha^*g_{\mathcal{M}}\text{-}$ O. Thus there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P}$ and $\mathbb{Q}$ such that $x \in \mathbb{P}$ but $y \notin \mathbb{P}$ and $y \in \mathbb{Q}$ but $x \notin \mathbb{Q}$. Hence $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1\text{Space}$.
- 3) Then each pair of distinct points x and y of $\mathbb{U}$ and $x \neq y$, there exists M-O sets $\mathbb{P}$ and $\mathbb{Q}$ such that $\mathbb{P} \cap \mathbb{Q} = \emptyset$ and $x \in \mathbb{P}$, $y \in \mathbb{Q}$. Since every M-O is $\alpha^*g_{\mathcal{M}}\text{-}$ O. Thus there exists a $\alpha^*g_{\mathcal{M}}\text{-}$ O sets $\mathbb{P}$ and $\mathbb{Q}$ such that $\mathbb{P} \cap \mathbb{Q} = \emptyset$ and $x \in \mathbb{P}$, $y \in \mathbb{Q}$. Hence $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2\text{Space}$.

Remark: 4.4 Vice versa of the results are not true.

Theorem: 4.5 A MTS $\mathbb{U}$ is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space if and only if for each pair of distinct points x and y of $\mathbb{U}$, $\alpha^*g_M\text{-cl}(x) \neq \alpha^*g_M\text{-cl}(y)$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a M-Topological Space. Let x and y be two distinct points of $\mathbb{U}$. Since $\mathbb{U}$ is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space, then there exists a $\alpha^*g_M\text{-O}$ set $\mathbb{P}$ such that $x \in \mathbb{P}$ and $y \notin \mathbb{P}$. Consequently, $\mathbb{P}^c$ is $\alpha^*g_M\text{-C}$ set containing y but not x . But $\alpha^*g_M\text{-cl}(y)$ is the smallest $\alpha^*g_M\text{-C}$ set containing y . Thus $y \in \alpha^*g_M\text{-cl}(y)$ but $x \notin \alpha^*g_M\text{-cl}(y)$ and $x \notin \mathbb{P}^c$. Hence $\alpha^*g_M\text{-cl}(x) \neq \alpha^*g_M\text{-cl}(y)$. **Conversely**, Suppose that for any two distinct points x and y of $\mathbb{U}$, $\alpha^*g_M\text{-cl}(x) \neq \alpha^*g_M\text{-cl}(y)$. Then there exists at least one point $z \in \mathbb{U}$ such that contained only in one of these two M-sets $\alpha^*g_M\text{-cl}(x)$ and $\alpha^*g_M\text{-cl}(y)$, say $\alpha^*g_M\text{-cl}(x)$. Suppose if $x \in \alpha^*g_M\text{-cl}(y)$, then $\alpha^*g_M\text{-cl}(x) \subseteq_M \alpha^*g_M\text{-cl}(y)$, since $\alpha^*g_M\text{-cl}(x)$ is the smallest $\alpha^*g_M\text{-C}$ set containing x . Thus $z \in \alpha^*g_M\text{-cl}(y)$, which is a contradiction to our assumption. Then $x \in [\alpha^*g_M\text{-cl}(y)]^c$, where $[\alpha^*g_M\text{-cl}(y)]^c$ is $\alpha^*g_M\text{-O}$ set containing x but not y . Hence $\mathbb{U}$ is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space.

Theorem: 4.6 In a M-Topological Space $\mathbb{U}$. Then $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_1$ Space if and only if singletons are $\alpha^*g_M\text{-C}$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Assume singletons $\{x\}$ and $\{y\}$ belong to $\alpha^*g_M\text{-C}$. Let x and y of $\mathbb{U}$. Now, $x \neq y$ implies $y \in \mathbb{U} - \{x\}$. Hence $\mathbb{U} - \{x\}$ is $\alpha^*g_M\text{-O}$ set containing y but not x . Similarly, we can prove $\mathbb{U} - \{y\}$ is $\alpha^*g_M\text{-O}$ set containing x but not y . Therefore $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_1$ Space. **Conversely**, let $\mathbb{U}$ be $\alpha^*g_M\text{-}\mathbb{T}_1$ Space and x any point of $\mathbb{U}$. Suppose $y \in \mathbb{U} - \{x\}$. Then $x \neq y$ and so there exist a M-O set $\mathbb{P}$ such that $x \notin \mathbb{P}$ and $y \in \mathbb{P}$. Consequently $y \in \mathbb{P} \subseteq_M \mathbb{U} - \{x\}$, that is $\mathbb{U} - \{x\} = \cup \{\mathbb{P} : y \in \mathbb{U} - \{x\}\}$ which is M-O. Thus $\{x\}$ is M-C. Since every M-C is $\alpha^*g_M\text{-C}$, then $\{x\}$ is $\alpha^*g_M\text{-C}$. Hence singletons are $\alpha^*g_M\text{-C}$.

Theorem: 4.7 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) Every $\alpha^*g_M\text{-}\mathbb{T}_2$ Space is a $\alpha^*g_M\text{-}\mathbb{T}_1$ Space.
- 2) Every $\alpha^*g_M\text{-}\mathbb{T}_1$ Space is a $\alpha^*g_M\mathcal{T}_{1/2}$ -Space.
- 3) Every $\alpha^*g_M\mathcal{T}_{1/2}$ -Space is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space.
- 4) Every $\alpha^*g_M\text{-}\mathbb{T}_1$ Space is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space.
- 5) Every $\alpha^*g_M\text{-}\mathbb{T}_2$ Space is a $\alpha^*g_M\text{-}\mathbb{T}_0$ Space.

Proof: 1) The proof is straightforward from the definitions.

2) The proof is obvious by theorem 4.6.

3) Let x and y be any two distinct points of $\mathbb{U}$. By theorem 4.6, the singleton set $\{x\}$ is M-C or M-O. (i) If $\{x\}$ is M-C, then $\mathbb{U} - \{x\}$ is M-O. So $y \in \mathbb{U} - \{x\}$ and $x \notin \mathbb{U} - \{x\}$. Therefore, we have $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_0$ Space. (ii) If $\{x\}$ is M-O, then $y \notin \{x\}$ and $x \in \{x\}$. Therefore, we have $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_0$ Space.

4) Suppose $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_1$ Space. Then for any two distinct points S and T in $\mathbb{U}$, there exists a $\alpha^*g_M\text{-O}$ sets $\mathbb{P}$ and $\mathbb{Q}$ such that $S \in \mathbb{P}$ but $T \notin \mathbb{P}$ and $S \notin \mathbb{Q}$ and $T \in \mathbb{Q}$. This implies there exists a $\alpha^*g_M\text{-O}$ sets $\mathbb{P}$ such that $S \in \mathbb{P}$ but $T \notin \mathbb{P}$. Hence $\mathbb{U}$ is $\alpha^*g_M\text{-}\mathbb{T}_1$ Space.

5) Then each pair of distinct points x and y of $\mathbb{U}$, $\alpha^*g_M\text{-O}$ sets $\mathbb{P}$ and $\mathbb{Q}$ such that one containing S but not T and the other containing T but not S . That is $x \in \mathbb{P}$ but $y \notin \mathbb{P}$ and

$y \in \mathbb{Q}$ but $x \notin \mathbb{Q}$. This implies there exists $\mathbb{P}$ in $\mathbb{U}$ such that $x \in \mathbb{P}$ but $y \notin \mathbb{P}$. Hence $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0$ Space.

Remark: 4.8 Vice versa of the results are not true.

Theorem: 4.9 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Then the following statements are equivalent:

- 1) $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2$ Space.
- 2) Let $x \in \mathbb{U}$ and For each $x \neq y$, there exist two disjoint $\alpha^*g_{\mathcal{M}}\text{-O}$ sets $\mathbb{P}$ containing x such that $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$.
- 3) For each $x \in \mathbb{U}$, $\cap\{\mathcal{M}\text{cl}(\mathbb{P})/\mathbb{P} \in \mu_{\mathcal{R}}(\mathbb{X}) \text{ and } x \in \mathbb{P}\} = \{x\}$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a M-Topological Space.

- 1) $\Rightarrow$ 2) Suppose $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2$ Space. Then there exist a M-O set $\mathbb{P}$ and $\mathbb{Q}$ containing x and y respectively. So $\mathbb{P} \subseteq_{\mathcal{M}} \mathbb{U} - \mathbb{Q}$. Therefore $\mathcal{M}\text{cl} \subseteq_{\mathcal{M}} \mathbb{U} - \mathbb{Q}$. So $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$.
- 2) $\Rightarrow$ 3) If possible for some $x \neq y$, we have $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$ for every M-O set $\mathbb{P}$ containing x , which then contradicts to 2). Hence $\cap\{\mathcal{M}\text{cl}(\mathbb{P})/\mathbb{P} \in \mu_{\mathcal{R}}(\mathbb{X}) \text{ and } x \in \mathbb{P}\} = \{x\}$.
- 3) $\Rightarrow$ 1) Let x and y of $\mathbb{U}$ and $x \neq y$. Then there exists a M-O set $\mathbb{P}$ containing x such that $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$. Let $\mathbb{Q} = \mathbb{U} - \mathcal{M}\text{cl}\{\mathbb{P}\}$. Then $y \in \mathbb{Q}$, $x \in \mathbb{P}$ and $\mathbb{P} \cap \mathbb{Q} = \emptyset$. Thus $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2$ Space.

Theorem: 4.10 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Then the following statements are equivalent:

- 1) $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.
- 2) Let $x \in \mathbb{U}$ and For each $x \neq y$, there exist two disjoint $\alpha^*g_{\mathcal{M}}\text{-O}$ sets $\mathbb{P}$ containing x such that $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$.
- 3) For each $x \in \mathbb{U}$, $\cap\{\mathcal{M}\text{cl}(\mathbb{P})/\mathbb{P} \in \mu_{\mathcal{R}}(\mathbb{X}) \text{ and } x \in \mathbb{P}\} = \{x\}$.

Proof: The proof the theorem follows similar approach as theorem 4.7 respectively.

Theorem: 4.11 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Then the following statements are equivalent:

- 1) $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0$ Space.
- 2) Let $x \in \mathbb{U}$ and For each $x \neq y$, there exist two disjoint $\alpha^*g_{\mathcal{M}}\text{-O}$ sets $\mathbb{P}$ containing x such that $y \notin \mathcal{M}\text{cl}\{\mathbb{P}\}$.
- 3) For each $x \in \mathbb{U}$, $\cap\{\mathcal{M}\text{cl}(\mathbb{P})/\mathbb{P} \in \mu_{\mathcal{R}}(\mathbb{X}) \text{ and } x \in \mathbb{P}\} = \{x\}$.

Proof: The proof the theorem 2) and 3) follows similar approach as theorem 4.7 respectively.

Theorem: 4.12 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a M-Topological Space.

- 1) Let $\mathbb{U}$ be a $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -Space. Then every M-subspace of a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0$ Space is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0$ Space.
- 2) Let $\mathbb{U}$ be a $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -Space. Then every M-subspace of a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.
- 3) Let $\mathbb{U}$ be a $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -Space. Then every M-subspace of a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2$ Space is

$\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2\text{Space}$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a M-Topological Space. Assume $\mathbb{V}$ be a M-subspace of a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, where $\mathbb{V}$ is a M-relative Topology of $\mathbb{U}$ on $\mathbb{V}$. Let x and y be two distinct points of $\mathbb{V}$. As $\mathbb{V} \subseteq_{\mathcal{M}} \mathbb{U}$ and $x, y \in \mathbb{U}$. Since $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then there exists a $\alpha^*g_{\mathcal{M}}\text{-O}$ set $\mathbb{P}$ such that $x \in \mathbb{P}$ and $y \notin \mathbb{P}$. Since $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}\text{-space}$, $\mathbb{P}$ is M-O in $\mathbb{U}$. Then by definition of Msubspace, $\mathbb{P} \cap \mathbb{V}$ is M-O in $\mathbb{V}$. This implies $\mathbb{P} \cap \mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ in $\mathbb{V}$ which contains x but not y . Hence $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.

The proof of the theorem 2) and 3) follows similar to approach as theorem 4.10.

Theorem: 4.13 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) If $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}\text{-irres}$ funct and $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.
- 2) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective and strongly $\alpha^*g_{\mathcal{M}}\text{-OM}$ and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$ then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{space}$.
- 3) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}\text{-OM}$ and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}\text{-space}$. If $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{space}$.
- 4) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}\text{-cont}$ bijective funct and $\mathbb{V}$ is $\mathcal{M}\text{-}\mathbb{T}_0\text{space}$, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

1) Let x and y be two distinct points of $\mathbb{V}$. Since $\mathbb{F}$ is a bijective, then there exists two distinct point $u, v \in \mathbb{U}$ such that $\mathbb{F}(u) = x$ and $\mathbb{F}(v) = y$. Since $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then there exists a $\alpha^*g_{\mathcal{M}}\text{-O}$ set $\mathbb{P}$ and $\mathbb{Q}$ such that $u \in \mathbb{P}$ but $v \notin \mathbb{P}$ and $v \in \mathbb{Q}$ but $u \notin \mathbb{Q}$. Therefore, $x = \mathbb{F}(u) \in \mathbb{F}(\mathbb{P})$ and $y = \mathbb{F}(v) \in \mathbb{F}(\mathbb{Q})$. Since $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-irres}$, then $\mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ in $\mathbb{U}$ such that $\mathbb{F}^{-1}(u) = x \in \mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(v) = y \notin \mathbb{F}^{-1}(\mathbb{P})$ or $\mathbb{F}^{-1}(u) = x \in \mathbb{F}^{-1}(\mathbb{Q})$ and $\mathbb{F}^{-1}(v) = y \notin \mathbb{F}^{-1}(\mathbb{Q})$. Thus for any two distinct points $x, y \in \mathbb{U}$, there exists $\mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ containing x but not y . Hence $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.

2) Let x and y be two distinct points of $\mathbb{V}$. Since $\mathbb{F}$ is a bijective, then there exists two distinct point $u, v \in \mathbb{U}$ such that $\mathbb{F}(u) = x$ and $\mathbb{F}(v) = y$. Since $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then there exists a $\alpha^*g_{\mathcal{M}}\text{-O}$ set $\mathbb{P}$ and $\mathbb{Q}$ such that $u \in \mathbb{P}$ but $v \notin \mathbb{P}$ and $v \in \mathbb{Q}$ but $u \notin \mathbb{Q}$. Therefore, $x = \mathbb{F}(u) \in \mathbb{F}(\mathbb{P})$ and $y = \mathbb{F}(v) \in \mathbb{F}(\mathbb{Q})$. Since $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-OM}$, then $\mathbb{F}(\mathbb{P})$ and $\mathbb{F}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ in $\mathbb{U}$ such that $\mathbb{F}^{-1}(u) = x \in \mathbb{F}(\mathbb{P})$ and $\mathbb{F}(v) = y \notin \mathbb{F}(\mathbb{P})$ or $\mathbb{F}(u) = x \in \mathbb{F}(\mathbb{Q})$ and $\mathbb{F}(v) = y \notin \mathbb{F}(\mathbb{Q})$. Thus for any two distinct points $x, y \in \mathbb{U}$, there exists $\mathbb{F}(\mathbb{P})$ and $\mathbb{F}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ containing x but not y . Hence $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.

3) Let x and y be two distinct points of $\mathbb{V}$. Since $\mathbb{F}$ is a bijective, then there exists two distinct point $u, v \in \mathbb{U}$ such that $\mathbb{F}(u) = x$ and $\mathbb{F}(v) = y$. Since $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$, then there exists a $\alpha^*g_{\mathcal{M}}\text{-O}$ set $\mathbb{P}$ and $\mathbb{Q}$ such that $u \in \mathbb{P}$ but $v \notin \mathbb{P}$ and $v \in \mathbb{Q}$ but $u \notin \mathbb{Q}$. Therefore, $x = \mathbb{F}(u) \in \mathbb{F}(\mathbb{P})$ and $y = \mathbb{F}(v) \in \mathbb{F}(\mathbb{Q})$. Since $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-OM}$, then $\mathbb{F}(\mathbb{P})$ and $\mathbb{F}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ in $\mathbb{U}$ such that $\mathbb{F}^{-1}(u) = x \in \mathbb{F}(\mathbb{P})$ and $\mathbb{F}(v) = y \notin \mathbb{F}(\mathbb{P})$ or $\mathbb{F}(u) = x \in \mathbb{F}(\mathbb{Q})$ and $\mathbb{F}(v) = y \notin \mathbb{F}(\mathbb{Q})$. Thus for any two distinct points $x, y \in \mathbb{U}$, there exists $\mathbb{F}(\mathbb{P})$ and $\mathbb{F}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}\text{-O}$ containing x but not y . Hence $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0\text{Space}$.

4) Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Since $\mathbb{F}$ is a bijective, then there exists two distinct point $u, v \in \mathbb{U}$ such that $\mathbb{F}(u)=x$ and $\mathbb{F}(v)=y$. Since $\mathbb{V}$ is $\mathcal{M}$ - $\mathbb{T}_0$ Space, then there exists a M-O set $\mathbb{P}$ and $\mathbb{Q}$ such that $u \in \mathbb{P}$ but $v \notin \mathbb{P}$ and $v \in \mathbb{Q}$ but $u \notin \mathbb{Q}$. Therefore, $x = \mathbb{F}(u) \in \mathbb{F}(\mathbb{P})$ and $y = \mathbb{F}(v) \in \mathbb{F}(\mathbb{P})$. Since $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}$ -cont, then $\mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}$ -O in $\mathbb{U}$ such that $\mathbb{F}^{-1}(u)=x \in \mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(v)=y \notin \mathbb{F}^{-1}(\mathbb{P})$ or $\mathbb{F}^{-1}(u)=x \in \mathbb{F}^{-1}(\mathbb{Q})$ and $\mathbb{F}^{-1}(v)=y \notin \mathbb{F}^{-1}(\mathbb{Q})$. Thus for any two distinct points $x, y \in \mathbb{U}$, there exists $\mathbb{F}^{-1}(\mathbb{P})$ and $\mathbb{F}^{-1}(\mathbb{Q})$ is $\alpha^*g_{\mathcal{M}}$ -O containing x but not y . Hence $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_0$ Space.

Theorem: 4.14 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) If $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}$ -irres funct and $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.
- 2) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective and strongly $\alpha^*g_{\mathcal{M}}$ -OM and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ space.
- 3) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}$ -OM and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space. If $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space, then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.
- 4) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ -cont bijective funct and $\mathbb{V}$ is $\mathcal{M}$ - $\mathbb{T}_1$ space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.

Proof: The proof the theorem 1),2),3) and 4) follows similar approach as theorem 4.13 respectively.

Theorem: 4.15 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) If $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}$ -irres funct and $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ Space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ Space.
- 2) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective and strongly $\alpha^*g_{\mathcal{M}}$ -OM and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ Space then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ space.
- 3) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a bijective, $\alpha^*g_{\mathcal{M}}$ -OM and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space. If $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ Space, then $\mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ space.
- 4) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ -cont bijective funct and $\mathbb{V}$ is $\mathcal{M}$ - $\mathbb{T}_2$ space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_2$ Space.

Proof: The proof the theorem 1),2),3) and 4) follows similar approach as theorem 4.13 respectively.

Theorem: 4.16 Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS.

- 1) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a strongly $\alpha^*g_{\mathcal{M}}$ -OM, bijective funct and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space, then $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.
- 2) A $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a $\alpha^*g_{\mathcal{M}}$ -cont, injective funct and $\mathbb{V}$ is $\mathcal{M}$ - $\mathbb{T}_1$ space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.
- 3) If $\mathbb{F}: \mathbb{U} \rightarrow \mathbb{V}$ is a injective, $\alpha^*g_{\mathcal{M}}$ -irres funct and $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space, then $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ - $\mathbb{T}_1$ Space.

Proof: The proof the theorem 2) and 3) follows similar approach as theorem 4.13 respectively.

Theorem: 4.17 If any point $x \in \mathbb{U}$ is contained only in the $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{U}$, then $\mathbb{U}$ cannot be $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Assume that $x \in \mathbb{U}$ is contained only in the $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{U}$. Suppose $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space. Then for any two points $x, y \in \mathbb{U}$, there exists $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{P}$ and $\mathbb{Q}$ such that $x \in \mathbb{P}$ but $y \notin \mathbb{P}$ and $x \notin \mathbb{Q}$ but $y \in \mathbb{Q}$. Since x is contained only in $\mathbb{U}$ and $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}$ -O, we have $\mathbb{P} = \mathbb{U}$, which is a contradiction to $y \notin \mathbb{P}$. Hence $\mathbb{U}$ cannot be $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.

Theorem: 4.18 If any point $x \in \mathbb{U}$ is contained in all non-empty $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{U}$, then $\mathbb{U}$ cannot be $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.

Proof: Let $(\mathbb{U}, \tau_{\mathcal{R}}(\mathbb{X}), \mu_{\mathcal{R}}(\mathbb{X}))$ be a MTS. Assume that $x \in \mathbb{U}$ is contained in all non-empty $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{U}$. Suppose $\mathbb{U}$ is a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space. Then for any two points $x, y \in \mathbb{U}$, there exists $\alpha^*g_{\mathcal{M}}$ -O set $\mathbb{P}$ and $\mathbb{Q}$ such that $x \in \mathbb{P}$ but $y \notin \mathbb{P}$ and $x \notin \mathbb{Q}$ but $y \in \mathbb{Q}$, which is a contradiction to $\mathbb{P}$ is a non-empty $\alpha^*g_{\mathcal{M}}$ -O set and hence assumption, $x \in \mathbb{P}$. Hence $\mathbb{U}$ cannot be $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space.

V: Conclusion

In this investigation, we looked into a few characteristics of $M\alpha^*$ -Generalized C Sets in MTS. Also we introduced a $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_0$ Space, $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_1$ Space and $\alpha^*g_{\mathcal{M}}\text{-}\mathbb{T}_2$.

Applications: MT can be applied in analyze the properties of graphs and the relationships between vertices, potentially leading to applications in network analysis and social network theory. MT finds application in various real world problems, particularly in areas like environmental analysis, medical applications and decision making. It may be analyzed the relationships between different factors and predict outcomes in situations where traditional methods may fall short. Future research in MT holds significant potential in various fields, including data analysis, computer science and material science.

References

- [1] Dr.P.Anbarasi Rodrigo, K.Selvakumari Prasha. Micro α^* -Generalized CF in MTS, Applied Mathematics, Automation And Computing, Taylor and Francis group as a proceeding book. (Scopus Indexed)
- [2] Dr.P.Anbarasi Rodrigo, K.Selvakumari Prasha. A New Class of Micro Generalized C Sets in MTS, AIP proceeding (communicated).
- [3] S.Chandrasekar, On MTS, Journal of new theory, 2019, 26: p. 23-31.
- [4] Anandhi.R and Balamani.N, Micro- α -Generalized C Sets in MTS. AIP Conference Proceeding, (2022), 2385, 130007.
- [5] P. Sathishmohan, G. Poongothai, K. Rajalakshmi, S. Stanley Roshan. Separation Axioms in Micro Semi Pre-Topological Spaces. Advances in Nonlinear Variational Inequalities, 2025, ISSN: 1092-910X.
- [6] Hariwan Z. Ibrahim, Micro Separation Axioms. International journal of analysis and Applications. Vol 18, N0.4, 2020, 572-585.
- [7] Padma1 , M. Saraswathi2 and A. Vadivel, Nano M separation axioms, Malaya Journal of Matematik, Vol. S, No. 1, 673-678, 2019.
- [8] K. S. Jenavee, r. Asokan and o. Nethaji, new separation axioms in $\mathfrak{Z}$ -nano topological spaces, turkish journal of computer and mathematics education vol.13 no.03 (2022), 133-139.
- [9] S Jackson, T Gnanapoo Denosha, P Mariappan, Separation Axioms through Nano JD open set , INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY, (2024_ pg. 9-13.